COMP3153/9153 Homework 0

Logic and Automata

Due: Fri 6th March 2020, 10am Submission guidelines are given at the end of this document.

Exercise 1 (Logics)

(50 Marks)

Question 1 The logical operator *nor*, denoted by \downarrow , is defined as

$$a \downarrow b = \neg (a \lor b)$$

Show that this operator is sufficient to define propositional logic. To do so, your task is to find formulas, consisting of variables and the operator \downarrow only, that are equivalent to the Boolean constants TRUE and FALSE, and the formulas $\neg a$, $a \lor b$, $a \land b$, and $a \Rightarrow b$.

Prove that your equivalences are correct.

Question 2 Three boxes are presented to you. One contains gold, the other two are empty. Each box has imprinted on it a clue as to its contents; the clues are:

Box 1: "The gold is not here"

Box 2: "The gold is not here"

Box 3: "The gold is in Box 2"

Only one message is true; the other two are false. Which box has the gold? Formalise the puzzle in propositional logic and find the solution using a truth table.



Figure 1: Automaton A

Exercise 2 (Automata)

(50 Marks)

Question 3 Let the alphabet be given by $\Sigma_A = \{a, b\}$. Describe formally, such as with a regular expression, the language accepted by automaton A above.

Question 4 Define an automaton B that accepts the following language:

 $\{w \mid w \in \{0,1\}^* \text{ and } w \text{ does not contain the string } 101\}$

Give both, the formal definition (5-tuple) as well as a schematic representation (similar to the one in Figure 1 of A).

Question 5 For given alphabet $\Sigma_C = \{a, b, e\}$, define an automaton C that accepts the following language:

 $\{\nu | \nu \in \Sigma_C^* \text{ and } \alpha \text{ only occurs in consecutive pairs in } \nu\}$

For example the words aa and bbaabaae should be accepted by the automaton, but not the words aaa or baaaae.

You can give the formal definition (5-tuple) or the schematic representation.

Question 6 Build the synchronised product (crossproduct) of A and C.

Submission Guidelines

- Due time: Fri 6th March 2020, 10am. No late submission allowed.
- Submit one PDF file (hw0.pdf) using the CSE give system by typing the command give cs3153 hw0 hw0.pdf on a CSE terminal. Alternatively use the online submission page.
- It is highly recommended that you use LATEX to prepare your document. A guide is provided on the course website.